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Decoupling of the Longitudinal Modes of Advanced Aircraft

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Nomenclature

 $K_f,\,t_f,\,K_e,\,t_e$ = gain and time constant of the flaperon and the elevator actuators m, I_{ν}, g = aircraft mass, y-axis moment of inertia, gravitational acceleration U, W, Q, Θ, Γ = x, z translation velocities, pitch rate, $(U_0, W_0, q_0, \vartheta_0, \gamma_0)$ pitch, and flight-path angle (nominal values) X, Z, M= x, z, and y external aerodynamic and propulsion forces and moment = dimensional stability derivatives $Z_i, M_i (i = u, \vartheta,$ $w, q, \dot{w}, \delta_f, \delta_e)$ $\delta_f(t), \delta_e(t),$ = flaperon and elevator deflections and $\delta_{cf}(t), \delta_{ce}(t)$ the respective pilot's commands $\vartheta(t), \gamma(t), q(t), w(t) = \text{pitch}, \text{ flight-path angle, pitch rate, and}$ vertical velocity increments

I. Introduction

 ${f T}$ HE coupling between the pilot commands (inputs) and the flight variables to be controlled (outputs) adversely affects the flying qualities. Flight variable mode decoupling with simultaneous satisfactory damping and settling time is one of the central problems in flight control. 1-6 Eigenstructure assignment and input output decoupling appear to be the most suitable design techniques satisfying the requirements for aircraft with many control effectors. According to the eigenstructure assignment technique, 1,2 after selecting an ideal set of closed-loop eigenvalues and corresponding eigenvectors, satisfactory mode decoupling and flying qualities are obtained.

Received Sept. 25, 1995; revision received April 17, 1996; accepted for publication April 17, 1996. Copyright © 1996 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved.

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In Refs. 4 and 5, using static and dynamic input-output decoupling controllers, independent control between the pitch angle and the downward speed has been performed. Other results in the field can be found in Ref. 3 for the case of an advanced fighter technology integration (AFTI) F-16 aircraft via robust eigenstructure assignment, in Ref. 6 where the model following technique is applied to decoupled flight control, and in Ref. 7 where decoupling and robustness is fulfilled via crossfeed, for the case of rotorcrafts.

The objective of this Note is to control independently the flightpath angle and pitch angle of a multimode aircraft, while preserving satisfactory flying qualities. To meet the benefits of both design techniques (eigenstructure assignment and input-output decoupling), the design technique of input-output decoupling with simultaneous arbitrary pole assignment is proposed. The problem is treated as a generic application facilitating the determination of the class of the stability derivatives for which pitch angle and flight-path angle can be controlled independently via static state feedback. The problem is proven to be solvable for almost all flight conditions, yielding exact decoupling, as well as desirable damping and settling time for the two resulting closed-loop subsystems. Each subsystem is a single-input-single-output all pole system having arbitrary denominator coefficients. These coefficients are the free parameters of the controller matrices. Appropriate tuning of these coefficients leads to adequate short period flying qualities for flying phase categories (A, B, or C). Using the present control scheme, the requirements of pitch pointing and vertical translation maneuvers can easily be met. Finally, all results are illustrated by simulation for an AFTI F-16 aircraft.

II. Model Description

The nonlinear equations describing the longitudinal motion of an aircraft are as follows8:

$$X - mg\sin(\Theta) = m(\dot{U} + QW)$$

$$Z + mg\cos(\Theta) = m(\dot{W} - QU), \qquad M = I_y \dot{Q} \qquad (1)$$

$$\dot{\Theta} = Q, \qquad \Theta - \Gamma = \tan^{-1}(W/U)$$

Here we study the longitudinal motion of an advanced aircraft, for straight symmetric flight with wings level. If W_0 is sufficiently small, the equality $\vartheta - \gamma \approx w/U_0$, can be used to derive the following short period approximation:

$$\dot{x}(t) = Ax(t) + Bu(t), \qquad y(t) = Cx(t), \qquad x(0-) = x_0$$
 (2a)

with

$$x(t) = [\gamma(t), q(t), \vartheta(t), \delta_f(t), \delta_e(t)]^T$$

$$y(t) = [\gamma(t), \vartheta(t)]^T, u(t) = [\delta_{cf}(t), \delta_{ce}(t)]^T (2b)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} \tilde{Z}_w & -\tilde{Z}_q/U_0 & -\tilde{Z}_w + (\tilde{g}/U_0)\sin(\vartheta_0) & -\tilde{Z}_{\delta f}/U_0 & -\tilde{Z}_{\delta e}/U_0 \\ -\tilde{M}_wU_0 & \tilde{M}_q & \tilde{M}_wU_0 + \tilde{M}_\vartheta & \tilde{M}_{\delta f} & \tilde{M}_{\delta e} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -t_f^{-1} & 0 \\ 0 & 0 & 0 & 0 & -t_e^{-1} \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ K_f t_f^{-1} & 0 \\ 0 & K_e t_e^{-1} \end{bmatrix}$$
(2c)

where $\tilde{M}_w = M_w + M_w \tilde{Z}_w$, $\tilde{M}_q = M_q + (U_0 + \tilde{Z}_q)M_w$, $\tilde{M}_\vartheta = -\tilde{g}M_w \sin(\vartheta_0)$, $\tilde{M}_{\delta e} = M_{\delta e} + M_w \tilde{Z}_{\delta e}$, $\tilde{M}_{\delta f} = M_{\delta f} + M_w \tilde{Z}_{\delta f}$,

$$ilde{Z}_w = rac{Z_w}{1 - Z_{\dot{w}}}, \qquad ilde{Z}_q = rac{Z_q + U_0 Z_{\dot{w}}}{1 - Z_{\dot{w}}}, \qquad ilde{Z}_{\delta e} = rac{Z_{\delta e}}{1 - Z_{\dot{w}}} \ ilde{Z}_{\delta f} = rac{Z_{\delta f}}{1 - Z_{\dot{w}}}, \qquad ilde{g} = rac{g}{1 - Z_{\dot{w}}} \ ilde{g}$$

The parameters Z_{ii} and M_{ii} are modeled to avoid confusion. For the preceding model the actuators dynamics are $\dot{\delta}_j(t) = -t_j^{-1}\delta_j(t) + t_j^{-1}K_j\delta_{cj}(t)(j=f,e)$.

III. Solvability Conditions

Here, it is determined under which conditions (over the aerodynamic and actuator parameters) the controller

$$u(t) = Fx(t) + G\omega(t)$$

$$\omega(t) = \begin{bmatrix} \gamma_c(t) \\ \vartheta_c(t) \end{bmatrix} \leftarrow \text{ flight-path angle external command}$$
 (3)
$$\leftarrow \text{ pitch angle external command}$$

results in a diagonally decoupled closed-loop system with outputs of the pitch angle and flight-path angle. As proven in Ref. 4, input-output decoupling is solvable if and only if the $\det[C^*B] \neq 0$, where

$$C^* = \begin{bmatrix} c_1 A^{d_1} \\ c_2 A^{d_2} \end{bmatrix}$$

$$d_i = \begin{cases} \min\{j : c_i A^j B \neq 0, \quad j = 0, 1, \dots, n-1\} \\ n-1 \quad \text{if} \quad c_i A^j B = 0, \quad \forall j \end{cases}$$

$$c_i : \text{ith row of } C$$

$$(4)$$

From Eqs. (2) and (4) we get

$$\det[C^*B] = \frac{K_f K_e (\tilde{Z}_{\delta e} \tilde{M}_{\delta f} - \tilde{Z}_{\delta f} \tilde{M}_{\delta e})}{t_f t_e U_0}$$

and $d_1 = 1$, $d_2 = 2$. Thus, the following result is established.

Theorem 1. Independent control of the pitch and the flight-path angle of the aircraft model (2), via static state feedback, is satisfied if and only if $\tilde{M}_{\delta e}\tilde{Z}_{\delta f} \neq \tilde{M}_{\delta f}\tilde{Z}_{\delta e}$. (The condition is true for almost all stability derivatives.⁸)

IV. Explicit Characterization of All Decoupling Controllers

To determine the set of all feedback matrices F and G solving the problem recall that $(d_1+1)+(d_2+1)=5=n=$ number of states. Thus, the design procedure developed in Ref. 4 can be used to yield the following explicit characterizations:

$$G = \frac{U_0}{\tilde{M}_{\delta e} \tilde{Z}_{\delta f} - \tilde{M}_{\delta f} \tilde{Z}_{\delta e}} \times \begin{bmatrix} -\tilde{M}_{\delta e} t_f K_f^{-1}(p_1)_0^{-1} & -\tilde{Z}_{\delta e} t_f K_f^{-1}(p_2)_0^{-1} U_0^{-1} \\ \tilde{M}_{\delta f} t_e K_e^{-1}(p_1)_0^{-1} & \tilde{Z}_{\delta f} t_e K_e^{-1}(p_2)_0^{-1} U_0^{-1} \end{bmatrix}$$
(5)

equilibrium (operating) point. For a maneuver involving more than one operating point, the values of the controller have to be renewed by lookup tables. This task can be carried out by an adjustment mechanism (in a real-time computer) also assigning the closed-loop poles. The explicitness of Eqs. (5) and (6) allows the adjustment mechanism to be executable in very sort time.

V. Decoupled Closed-Loop System

Using Eqs. (2), (5), and (6) the general form of the decoupled closed-loop transfer function matrix is computed to be

$$C(sI - A - BF)^{-1}BG$$

$$= \begin{bmatrix} \frac{(p_1)_0^{-1}}{s^2 + s(\beta_1)_1 + (\beta_1)_0} & 0 \\ 0 & \frac{(p_2)_0^{-1}}{s^3 + s^2(\beta_2)_2 + s(\beta_2)_1 + (\beta_2)_0} \end{bmatrix}$$
(10)

The coefficients $(\beta_i)_j$ are related to the arbitrary parameters $(\lambda_i)_j$ as follows:

$$\begin{bmatrix} (\beta_1)_1 \\ (\beta_1)_0 \end{bmatrix} = \begin{bmatrix} -\tilde{M}_q - \tilde{Z}_w \\ \tilde{M}_q \tilde{Z}_w - \tilde{M}_w \tilde{Z}_q \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & -\tilde{Z}_w \end{bmatrix} \begin{bmatrix} (\lambda_1)_1 \\ (\lambda_1)_2 \end{bmatrix}$$
(11a)

$$\begin{bmatrix} (\beta_2)_2 \\ (\beta_2)_1 \\ (\beta_2)_0 \end{bmatrix} = \begin{bmatrix} -\tilde{M}_q - \tilde{Z}_w \\ \tilde{M}_q \tilde{Z}_w - \tilde{M}_\vartheta - \tilde{M}_w (U_0 + \tilde{Z}_q) \\ \tilde{g} \tilde{M}_w \sin(\vartheta_0) + \tilde{M}_\vartheta \tilde{Z}_w \end{bmatrix}$$

$$+ \begin{bmatrix} 1 & 0 & 0 \\ -\tilde{M}_q & -1 & 0 \\ -\tilde{M}_{\vartheta} - U_0 \tilde{M}_w & 0 & -1 \end{bmatrix} \begin{bmatrix} (\lambda_2)_1 \\ (\lambda_2)_2 \\ (\lambda_2)_3 \end{bmatrix}$$
(11b)

For every choice of $[(\beta_1)_0 \ (\beta_1)_1 \ (\beta_2)_0 \ (\beta_2)_1 \ (\beta_2)_2]$ there exist (unique) $[(\lambda_1)_2 \ (\lambda_1)_1 \ (\lambda_2)_3 \ (\lambda_2)_2 \ (\lambda_2)_1]$ satisfying Eqs. (11). Thus, the general form of $C(sI-A-BF)^{-1}BG$ has five arbitrary poles. Combination of this remark with Theorem 1 and the fact that the model (2) is of order five yields the following.

Theorem 2. If the condition of Theorem 1 is satisfied, then, independent control of the pitch angle and the flight-path angle with simultaneously arbitrary pole assignment can always be achieved.

VI. Simulation Results

Consider an AFTI F-16 (Ref. 1) with $Z_w=-1.3411, Z_{\dot{w}}=0,$ $Z_{\delta f}/U_0=-0.25183, Z_{\delta e}/U_0=-0.16897, \tilde{M}_g=-0.86939,$ $U_0\tilde{M}_w=43.223, \, \tilde{M}_\vartheta=0, \, \tilde{M}_{\delta e}=-17.251, \, \tilde{M}_{\delta f}=-1.5766,$

$$F = \frac{U_0}{\tilde{M}_{\delta e}\tilde{Z}_{\delta f} - \tilde{M}_{\delta f}\tilde{Z}_{\delta e}} \begin{bmatrix} -\tilde{M}_{\delta e}t_fK_f^{-1} & -\tilde{Z}_{\delta e}t_fK_f^{-1}U_0^{-1} \\ \tilde{M}_{\delta f}t_eK_e^{-1} & -\tilde{Z}_{\delta f}t_eK_e^{-1}U_0^{-1} \end{bmatrix} \begin{bmatrix} (\lambda_1)_1 & (\lambda_1)_2\tilde{Z}_qU_0^{-1} - \tilde{g}\sin(\vartheta_0)U_0^{-1} + \tilde{Z}_w & f_{13} & f_{14} & f_{15} \\ (\lambda_2)_1U_0\tilde{M}_w & (\lambda_2)_2 & (\lambda_2)_3 & f_{24} & f_{25} \end{bmatrix}$$
(6)

where

$$f_{13} = (\lambda_{1})_{2} \left[\tilde{Z}_{w} - U_{0}^{-1} \tilde{g} \sin(\vartheta_{0}) \right] - \tilde{M}_{q} \tilde{Z}_{w} + \tilde{M}_{w} \tilde{Z}_{q}$$

$$+ U_{0}^{-1} \tilde{M}_{\vartheta} \tilde{Z}_{q} + U_{0}^{-1} \tilde{g} \tilde{M}_{q} \sin(\vartheta_{0})$$

$$f_{14} = (\lambda_{1})_{2} U_{0}^{-1} \tilde{Z}_{\delta f} - U_{0}^{-1} t_{f}^{-1} \tilde{Z}_{\delta f} + U_{0}^{-1} (\tilde{M}_{\delta f} \tilde{Z}_{q} - \tilde{M}_{q} \tilde{Z}_{\delta f})$$

$$(8)$$

$$f_{15} = (\lambda_{1})_{2} U_{0}^{-1} \tilde{Z}_{\delta e} - U_{0}^{-1} t_{e}^{-1} \tilde{Z}_{\delta e} + U_{0}^{-1} (\tilde{M}_{\delta e} \tilde{Z}_{q} - \tilde{M}_{q} \tilde{Z}_{\delta e})$$

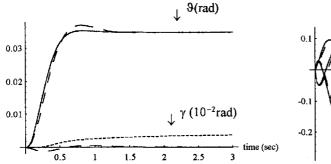
$$f_{24} = -(\lambda_{2})_{1} \tilde{M}_{\delta f} + t_{f}^{-1} \tilde{M}_{\delta f} + \tilde{M}_{\delta f} \tilde{Z}_{w} - \tilde{M}_{w} \tilde{Z}_{\delta f}$$

$$f_{25} = -(\lambda_{2})_{1} \tilde{M}_{\delta e} + t_{e}^{-1} \tilde{M}_{\delta e} + \tilde{M}_{\delta e} \tilde{Z}_{w} - \tilde{M}_{w} \tilde{Z}_{\delta e}$$

$$(9)$$

and $(p_i)_0^{-1}$ and $(\lambda_i)_j$ are arbitrary parameters. Relations (5) and (6) are explicit formulas implementable by elementary operations upon the stability derivatives, the actuators' parameters, and the nominal values ϑ_0 and U_0 . The matrices F and G depend on the parameters of the aircraft model (2), which is linearized around an

 $Z_q/U_0=-0.00665$, $t_f^{-1}=t_e^{-1}=20$, and $K_e=K_f=1$. The variables $q,\vartheta,\gamma,\delta_e$, and δ_f are in radians or radians per second. The condition of Theorem 1 is satisfied. Substituting in Eqs. (5) and (6) the choices $(\lambda_1)_1=6.48142$, $(\lambda_1)_2=18.2895$, $(\lambda_2)_1=28.0075$, $(\lambda_2)_2=-279.578$, and $(\lambda_2)_3=-2141.8$, the closed-loop poles are at -1, -19, -19.5, and $-5.609\pm4.19i$. Choosing $(p_1)_0^{-1}=19.5$ and $(p_2)_0^{-1}=931.32$, the responses (state) of the closed-loop system for pitch pointing ($\vartheta_e=2$ deg = 0.0349 rad, $\gamma_e=0$) are shown in Fig. 1. As is shown, the state vector performance is quite satisfactory because the rising time of the pitch angle is very short and the flight-path angle is zero. Analogous results can be derived for vertical translation. The controller [Eqs. (5) and (6)] is tested for large uncertainties on the elements of A involving aerodynamic parameters. Perturbations over 25% of the nominal values are considered. The system performance is shown in Fig. 1 (pitch pointing) where it is observed that the rising time of the pitch angle remains short, whereas the deviations of the attitude and the positions are sufficiently small.



 $\delta_e(\text{rad})$ q(rad/sec) $0.5 \quad 1 \quad 1.5 \quad 2 \quad 2.5 \quad 3 \quad \text{time (sec)}$ $0.2 \quad \delta_f(\text{rad})$

Fig. 1 Responses for pitch pointing: ——, nominal case; ----, nonlinear case; and ——, perturbed (uncertain) case.

To check the performance of the controller via nonlinear simulation, we substitute $Q=q+q_0, \ \dot{Q}=\dot{q}+\dot{q}_0, \ Z=\delta Z+Z_0, \ M=\delta M+M_0, \ \Theta=\vartheta+\vartheta_0, \ \Gamma=\gamma+\gamma_0, \ W=w+W_0, \ \text{and} \ \dot{W}=\dot{w}+\dot{W}_0$ in Eq. (1). Then Eq. (1) can be approximated for a steady flight (constant altitude and velocity) by the following nonlinear (for ϑ and γ) model:

$$\tilde{Z}_{w} \left[\tan(\vartheta - \gamma + \vartheta_{0} - \gamma_{0}) - \frac{W_{0}}{U_{0}} \right] + \frac{\tilde{Z}_{q} + U_{0}}{U_{0}} q
+ \frac{\tilde{Z}_{\delta_{e}}}{U_{0}} \delta_{e} + \frac{\tilde{Z}_{\delta_{f}}}{U_{0}} \delta_{f} - \frac{\tilde{g}[\cos(\vartheta_{0}) - \cos(\vartheta + \vartheta_{0})]}{U_{0}}
= \frac{\dot{\vartheta} - \dot{\gamma}}{\cos^{2}(\vartheta - \gamma + \vartheta_{0} - \gamma_{0})}$$
(12)

$$\tilde{M}_{w}[-W_{0} + U_{0} \tan(\vartheta - \gamma + \vartheta_{0} - \gamma_{0})] + \tilde{M}_{q}q
- M_{w}\tilde{g}[\cos(\vartheta_{0}) - \cos(\vartheta + \vartheta_{0})] + \tilde{M}_{\delta_{e}}\delta_{e} + \tilde{M}_{\delta_{f}}\delta_{f} = \dot{q}
\dot{\vartheta} = q$$
(13)

where the approximations $\delta Z/m = Z_w w + Z_q q + Z_{\delta_e} \delta_e + Z_{\delta_f} \delta_f + Z_{\dot{w}} \dot{w}$ and $\delta M/I_y = M_w w + M_q q + M_{\delta_e} \delta_e + M_{\delta_f} \delta_f + M_{\dot{w}} \dot{w}$ and the assumption that the actuators are described by linear models have been used. For deriving simulation results, the trim points (initial conditions) of the model for steady flight are considered to be $\vartheta_0 = \gamma_0 = 0$ and $W_0 = 0$.

To the nonlinear system [Eqs. (12) and (13)] apply the feedback law in Eqs. (5) and (6). The resulting closed-loop system is close to being decoupled with satisfactory performance. For pitch pointing the closed-loop performance appears to be (visually) the same with that of the linear case (Fig. 1). A slight difference of about 10^{-5} rad is observed for the flight-path angle.

VII. Conclusions

The pitch and flight-path angle of a multimode aircraft have been independently controlled, via static state feedback, using the input-output decoupling with simultaneous arbitrary pole assignment technique. The satisfactory closed-loop performance has been illustrated by simulation for an AFTI F-16 aircraft. However, the present technique basically differs from that of eigenstructure assignment on the starting point, which for the present case is exact decoupling and not specification of the desired eigenvalues. Based on this difference the following results have been derived: the set of stability derivatives for which exact decoupling is satisfied (Theorem 1), the set of all decoupling controllers (in terms of stability derivatives and free parameters), and the decoupled closed-loop transfer function with arbitrary poles and gains. After appropriate evaluation of these free elements, the desirable damping and settling time are precisely obtained.

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Global Stabilization of Flexible Multibody Spacecraft Using Quaternion-Based Nonlinear Control Law

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Introduction

R LEXIBLE multibody space systems are characterized by highly nonlinear dynamics, significant elastic motion with low inherent damping, and uncertainties in the mathematical model. Global asymptotic stability of multibody flexible space systems controlled by a nonlinear dissipative controller was recently established. However, the controller was restricted to have a scalar gain for quaternion feedback. Subsequently, robust nonlinear attitude control of a rigid spacecraft with nonlinear rotational dynamics was investigated using a quaternion feedback control law with a more general structure. The objective of this Note is to generalize the results of Ref. 1 to include a broader class of nonlinear controllers. It can also be considered as the generalization of Ref. 2 from single-body rigid

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